DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING, UCLA
ECE 239AS: COMPUTATIONAL IMAGING

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PSET 2 : PINHOLE CAMERAS

SL. NO.	TOPIC	MAX. POINTS	GRADED POINTS	REMARKS
1	Ray diagram	01		
2	Analysis	01		
3	Real pinhole camera	01		
4	Camera construction	02		
5	Verifying the camera	01		
6	Experimental analysis	01		
7	Fourier analysis- ideal	01		
8	Fourier analysis- real	01		
9	Analysis	01		
	Total	10		

1 Motivation

The simplest camera that one can think of is a pinhole camera. In such a camera, a tiny hole/orifice at the front acts as the aperture, through which the light enters. The back of the camera behaves as the screen on which the image is projected. Unlike conventional cameras that we are used to, a pinhole camera setup has no lens system for focusing the light rays incident on the system. In this problem set, we understand the theory behind pinhole cameras, as well as work towards building one.

2 Pinhole Camera Basics

Before building our camera, we look at some theory, to try and understand what we can expect from the camera. Let us consider a pinhole camera with the following top view:

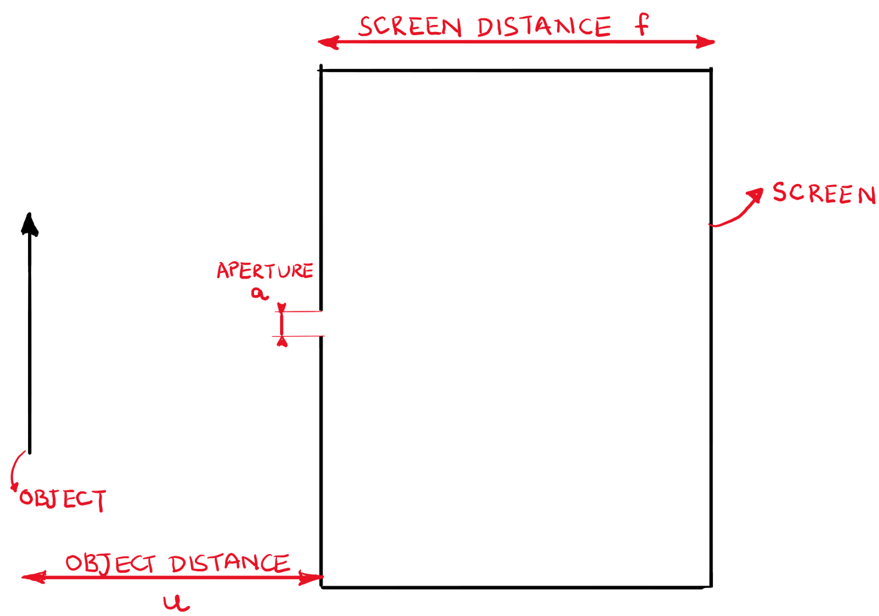

Figure 1: Top view of a simple pinhole camera.

Q1) Please draw the ray diagram for the image formation process in a pinhole camera. For this experiment, assume that the aperture a on the camera is sufficiently small (this is called an ideal pinhole camera). You can draw it by hand on a piece of paper, and add an image in the box below. Remember to mark out relevant dimensions in the diagram. (1 points)

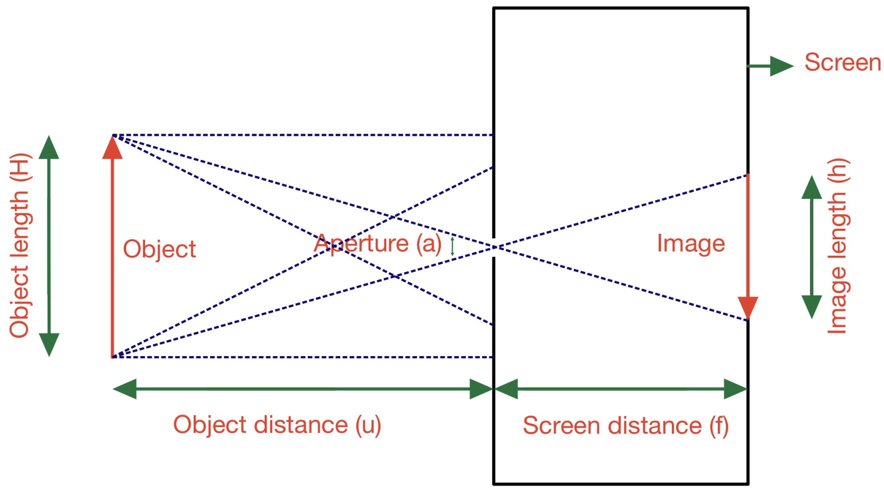

Figure 2: Ray diagram for image formation in an ideal pinhole camera.

Q2) What is the size (length) of the image created by the camera? Also report the absolute value of the magnification factor of the camera, in terms of the dimensions in the ray diagram. Note that this is defined as $m = \frac{\text{length of the image}}{\text{length of the object}}$. How can this magnification factor be increased/decreased? (1 points)

For an object with size (length) H, placed at a distance of u from the pinhole camera, an image of size (length) h is formed on the screen which is placed at a distance f units away from the pinhole behind it. Using these values, the image size can be represented by:

$$h = \frac{Hf}{u} \quad (1)$$

From the above equation, the magnification factor m is given by:

$$m = \frac{h}{H} = \frac{f}{u} \quad (2)$$

This magnification factor m can be increased by either increasing screen distance f or by reducing object distance u. It can be decreased by either decreasing screen distance f or by increasing object distance u.

Q3) Let us now look at what happens if the aperture has a finite size. Consider a finite aperture size of a, as shown in the ray diagram. From your understanding of pinhole cameras, what do you expect to happen to the image? No need to draw a ray diagram here- just a couple of sentences on how the image changes. (1 points)

A pinhole camera with a finite aperture a will be less effective as compared to the one with an infinitesimally small hole. This is because, each point on the image plane (screen) will receive light rays coming from multiple points on the object, thus resulting in a blurry and hazy, overlapped image.

3 Building a Pinhole Camera

Now that we understand how a pinhole camera works, the next step is to build a simple camera to see it in action!

Q1) Follow the steps here [1] for the 'without filters' pinhole camera (first part of the tutorial on the webpage) to create your own! For our experiments, we will build a camera with a small aperture, as well as a hole next to it so that you can use a camera (e.g. your cell phone camera) to capture the image formed. Note that while using the camera, the hole for it should be completely covered so that no light leaks into the setup.
Post an image of your camera setup in the box below. (2 points)

Figure 3: Pinhole camera setup.

Q2) Add an image of a scene from your camera, as well as an image of the same scene from your pinhole camera in the box below. What are some of the properties of the image that you observe? (1 points)

Figure 4: (Left) Image of the scene captured with the pinhole camera with a very small aperture. (Right) Image of the same scene when captured with a smartphone camera.

The image captured by the pinhole camera is vertically inverted, has lesser brightness and provides lesser features as compared to the smartphone camera image. Further, the pinhole camera can produce images only when the scenes are very brightly illuminated.

Q3) Let us now try to experimentally verify our observations from Q3, Part 1. Seal the original pinhole with some tape (ensure that the tape does not let light through) and create a hole with a larger diameter next to it. Now, capture an image of the same scene as the previous part, with this new aperture, and post the image in the box below. How is this image different from the one with the smaller aperture? (1 points)

Figure 5: Image captured via the same camera with bigger aperture.

Compared to the image captured with a smaller aperture, this image is more blurry, hazy and several spatial features and intensity variations are lost.

4 Frequency Analysis of Imaging Apertures

Having understood and built a pinhole camera, this part is dedicated to applying frequency analysis tools from the class to this system.

Q1) What is the transfer function (in the frequency domain) of the ideal pinhole camera system (infinitesimally small aperture)? (1 points)

The transfer function of an imaging system is called the Optical transfer function (OTF) which may be complex-valued. Hence the Modulation transfer function (MTF), which is the absolute magnitude of the complex-valued OTF is defined as the Fourier transform of the point spread function (PSF) - which is the 2-D impulse response of a point source imaging system. In our case, the pinhole is the point source and hence the PSF can be defined as:

$$psf(x,y) = \delta(x,y) = \begin{cases} 1, & \sqrt{x^2+y^2} = 0 \\ 0, & \sqrt{x^2+y^2} > 0 \end{cases} \tag{3}$$

where $\delta(x,y)$ is the 2-D dirac delta function whose value is 1 when $x = y = 0$ and 0 otherwise. This function accurately models the ideal pinhole camera where the aperture is infinitesimally small ($a \to 0$). Then the Fourier transform of this $psf(x,y)$ is given by:

$$H(\omega_x, \omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x,y)e^{-j(\omega_x x+\omega_y y)}dxdy \tag{4}$$

substituting $x = 0$ and $y = 0$ gives:

$$H(\omega_x, \omega_y) = 1 \tag{5}$$

We obtain an MTF with constant response of 1. This transfer function is frequency independent. Hence, this system has uniform constant response across all frequencies ω_x and ω_y.

Q2) Now, assume the pinhole aperture is a square with a finite side length a. What is the transfer function of this system? Use an appropriate 2D coordinate system to make calculations simpler. (1 points)

Assuming that the pinhole camera now has a finite square aperture of side size a, centered at the origin, the input function is now:

$$square(x,y) = \begin{cases} 1, & |x| \leq a/2, |y| \leq a/2 \\ 0, & otherwise \end{cases} \tag{6}$$

So the Fourier transform is given by:

$$H_a(\omega_x, \omega_y) = \int_{-a/2}^{a/2}\int_{a/2}^{a/2} e^{-j(\omega_x x+\omega_y y)}dxdy \tag{7}$$

which can be separated as:

$$H_a(\omega_x, \omega_y) = \int_{-a/2}^{a/2} e^{-j\omega_x x}dx. \int_{a/2}^{a/2} e^{-j\omega_y y}dy \tag{8}$$

$$\implies H_a(\omega_x, \omega_y) = \left[\frac{e^{-j\omega_x x}}{-j\omega_x}\right]_{-a/2}^{a/2} \cdot \left[\frac{e^{-j\omega_y y}}{-j\omega_y}\right]_{-a/2}^{a/2} \tag{9}$$

$$\implies H_a(\omega_x, \omega_y) = \left[\frac{e^{-\frac{ja\omega_x}{2}} - e^{\frac{ja\omega_x}{2}}}{-j\omega_x}\right] \cdot \left[\frac{e^{-\frac{ja\omega_y}{2}} - e^{\frac{ja\omega_y}{2}}}{-j\omega_y}\right] \tag{10}$$

$$\implies H_a(\omega_x, \omega_y) = \left[\frac{2sin(0.5a\omega_x)}{\omega_x}\right]\left[\frac{2sin(0.5a\omega_y)}{\omega_y}\right] \tag{11}$$

This results in:

$$H_a(\omega_x, \omega_y) = a^2 sinc(0.5a\omega_x)sinc(0.5a\omega_y) \tag{12}$$

which is the Fourier transform of a finite aperture pinhole camera system. The transfer function is a product of 2 *sinc* functions - one for each dimension x and y.

Q3) Which of the two apertures in the previous two questions will capture a sharper image? Please justify your answer numerically from the transfer functions you have calculated from both apertures? It may be helpful to read up on a concept known as "Modulation Transfer Function 50" (MTF50). (1 points)

From equation (5), we can see that for an ideal pinhole camera:

$$\lim_{\omega_x \to \infty} |\, H(\omega_x, \omega_y)\, | = \lim_{\omega_y \to \infty} |\, H(\omega_x, \omega_y)\, | = 1 \tag{13}$$

and for the one with a finite aperture a, from equation (12):

$$\lim_{\omega_x \to \infty} |\, H_a(\omega_x, \omega_y)\, | = \lim_{\omega_y \to \infty} |\, H_a(\omega_x, \omega_y)\, | = 0 \tag{14}$$

This is because, for the *sinc* function, as $\omega_x \to \infty$, $sinc(0.5a\omega_x) \to 0$. So due to this nature of the MTF $|H_a(\omega_x, \omega_y)|$, all high spatial-frequency components are suppressed and the image will only possess low spatial-frequency components making the finite aperture pinhole camera capture a blurry image. On the contrary, the ideal pinhole camera has uniform and constant response across all frequencies. Hence it captures a sharper image.

References

[1] [Online]. Available: https://alecs590.wordpress.com/assignment-5-pinhole-and-anti-pinhole/